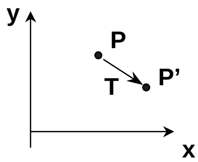
Transformações geométricas

- **translação**



xp' = xp + dx

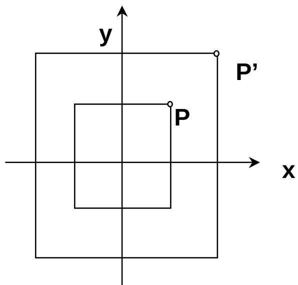
yp' = yp + dy

P = [xp yp]
P'= [xp' yp']

T = [dx dy]

[xp' yp'] = [xp yp] + [dx dy]

- **Mudança de escala**



xp' = s xp

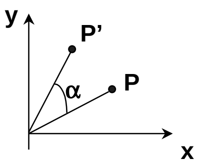
yp' = s yp

P = [xp yp]
P'= [xp' yp']

fator = s

$$[xp'\ yp'] = [xp\ \ yp] \cdot \begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix}$$

- **Rotação**



xp' = xp cos α - yp sen α

yp' = xp sen α + yp cos α

P = [xp yp]
P'= [xp' yp']

ângulo = α

$$[xp'\ yp'] = [xp\ \ yp]\ .\ \begin{bmatrix} \cos\alpha & \text{sen}\,\alpha \\ -\text{sen}\,\alpha & \cos\alpha \end{bmatrix}$$

Representação matricial das transformações 2D

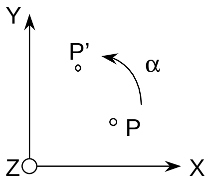
Escala: $[xp'\ yp'\ 1] = [xp\ \ yp\ 1]\ .\ \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rotação: $[xp'\ yp'\ 1] = [xp\ \ yp\ 1]\ .\ \begin{bmatrix} \cos\alpha & \text{sen}\,\alpha & 0 \\ -\text{sen}\,\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Tanslação: $[xp'\ yp'\ 1] = [xp\ yp\ 1]\ .\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ dx & dy & 1 \end{bmatrix}$

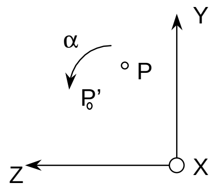
E se o ponto for representado por matriz coluna?

❖ Rotação



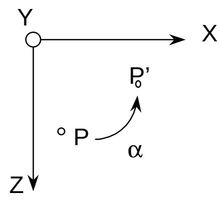
em torno de Z

$$\begin{bmatrix} \cos\alpha & \text{sen}\,\alpha & 0 \\ -\text{sen}\,\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



em torno de X

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \text{sen}\,\alpha \\ 0 & -\text{sen}\,\alpha & \cos\alpha \end{bmatrix}$$



em torno de Y

$$\begin{bmatrix} \cos\alpha & 0 & -\text{sen}\,\alpha \\ 0 & 1 & 0 \\ \text{sen}\,\alpha & 0 & \cos\alpha \end{bmatrix}$$

Escala: $[xp'\ yp'\ zp'\ 1] = [xp\ \ yp\ \ zp\ \ 1] \cdot \begin{bmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Tanslação: $[xp'\ yp'\ zp'\ 1] = [xp\ yp\ zp\ 1] \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ dx & dy & dz & 1 \end{bmatrix}$

Representação matricial (3D)

Rotação em torno de Z:

$$[xp' \; yp' \; zp' 1] = [xp \;\; yp \;\; zp \; 1] \; . \begin{bmatrix} \cos\alpha & \text{sen}\,\alpha & 0 & 0 \\ -\,\text{sen}\,\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotação em torno de Y:

$$[xp' \; yp' \; zp' \; 1] = [xp \;\; yp \; zp \; 1] \; . \begin{bmatrix} \cos\alpha & 0 & -\text{sen}\,\alpha & 0 \\ 0 & 1 & 0 & 0 \\ \text{sen}\,\alpha & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotação em torno de X:

$$[xp' \; yp' \; zp' 1] = [xp \;\; yp \;\; zp \; 1] \; . \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & \text{sen}\,\alpha & 0 \\ 0 & -\,\text{sen}\,\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$